Coherence time of a Bose-Einstein condensate

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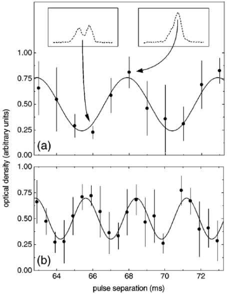
Phase coherence of a BEC

First measurement of time coherence (JILA 1998):

$\pi/2$-**pulses** between **two internal states**

$$\begin{array}{ccccccc} \phi_a & & \frac{\phi_a}{\sqrt{2}} & & \frac{\phi_a}{\sqrt{2}} e^{i\theta_a} & & \frac{1}{2}(\phi_a e^{i\theta_a} + \phi_b e^{i\theta_b}) \\ & \rightarrow & & \rightarrow & & \rightarrow & \\ 0 & & \frac{\phi_a}{\sqrt{2}} & & \frac{\phi_b}{\sqrt{2}} e^{i\theta_b} & & \frac{1}{2}(\phi_a e^{i\theta_a} - \phi_b e^{i\theta_b}) \end{array}$$



Main aim:

How temperature determines the fundamental limit to phase coherence?

Considered system

- the Bose condensed gas at thermal equilibrium
- total number of particles is fixed to N
- small non condensed fraction
- 3D spatially homogeneous system with periodic boundary conditions

At $T = 0$ correlation function $\langle a_0^\dagger(t) a_0 \rangle$ oscillates with frequency μ, **Beliaev 1958**. **No phase spreading.**

We expect that **at $T \neq 0$ interactions** with non condensed modes (Bogoliubov excitations) **perturb the condensate phase.**

Numerical experiment

We describe the condensate and the thermal fraction using the classical fields method.

- **advantages:** classical fields can be simulated exactly and contain the full non-linear dynamics
- **UV catastrophe:** $n_{\mathbf{k}}^{\mathrm{cl}} = \frac{k_B T}{\epsilon_{\mathbf{k}}} \neq \frac{1}{e^{\beta \epsilon_{\mathbf{k}}} - 1}$ cut-off is needed (results depend on cut-off)

A. Sinatra *et al*, PRL **87**, 210404 (2001); M.J. Davis *et al*, PRL **87**, 160402 (2001); M. Brewczyk *et al*, J.Phys.B **40**, R1 (2007)

At thermal equilibrium

Natural basis – plane waves:

$$\psi(\mathbf{r},t) = \sum_{\mathbf{k}} a_{\mathbf{k}} e^{i\mathbf{kr}}$$

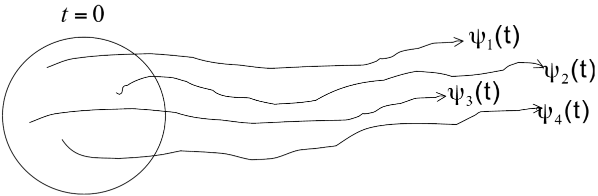
- **the condensate:**

$$a_0 = e^{i\theta}\sqrt{N_0}$$

Observable of interest

$\mathrm{Var}\,\varphi(t) = \langle \varphi(t)^2 \rangle - \langle \varphi(t) \rangle^2$ where $\varphi(t) = \theta(t) - \theta(0)$

Classical fields simulations



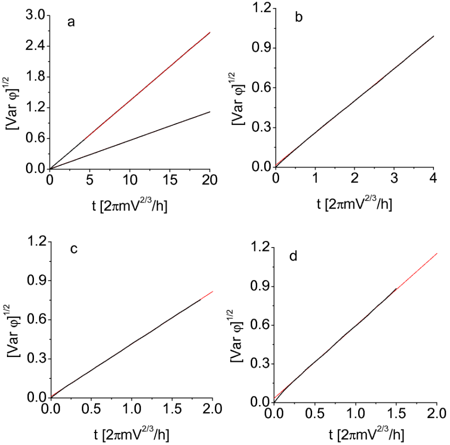
- at $t = 0$ generate ensemble of fields $\psi_j(\mathbf{r}, t)$
- evolve with NLSE:

$$i\hbar\dot{\psi}_j(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m}\Delta + g|\psi_j(\mathbf{r}, t)|^2\right)\psi_j(\mathbf{r}, t)$$

- calculate observables (**example:** average number of condensed atoms $\langle N_0 \rangle = \frac{1}{N_{\text{real}}}\sum_j N_0^{(j)}$)

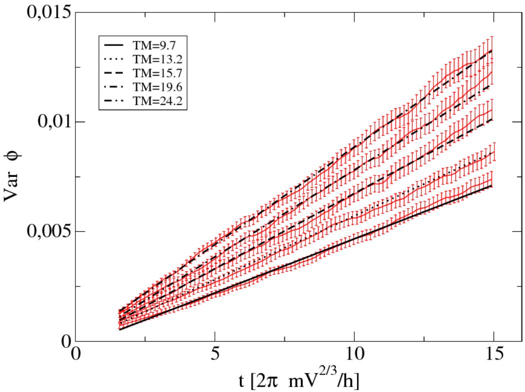
Initial state in canonical ensemble

Classical fileds generated for a given temperature T and number of atoms N. **Fluctuations of energy are present in the field samples.**



Initial state in microcanonical ensemble

Classical fileds generated for a **given energy E** and **number of atoms N**.



For convenience, we parametrize the microcanonical ensemble by the temperature T such that the mean energy in the canonical ensemble at temperature T is equal to E.

Result

Variance of the phase depends strongly on the ensemble!

In the canonical: ballistic expansion $\mathrm{Var}\,\varphi \sim t^2$.

In the microcanonical: diffusive motion $\mathrm{Var}\,\varphi \sim t$.

Analytical approach

Split the field operator $\hat{\psi}$ into condensate + non condensed parts:

$$\hat{\psi}(\mathbf{r}, t) = \frac{\hat{a}_0}{\sqrt{V}} + \hat{\psi}_\perp$$

the non-condensed part expanded over the Bogoliubov modes:

$$\hat{\psi}_\perp(\mathbf{r}, t) = e^{-i\hat{\theta}} \sum_{\mathbf{k}\neq 0} (U_\mathbf{k} \hat{b}_\mathbf{k}(t) + V_\mathbf{k} \hat{b}^\dagger_{-\mathbf{k}}(t)) \frac{e^{i\mathbf{kr}}}{\sqrt{V}}$$

the condensate $\hat{a}_0 = e^{i\hat{\theta}} \sqrt{\hat{N}_0}$

Observable of interest

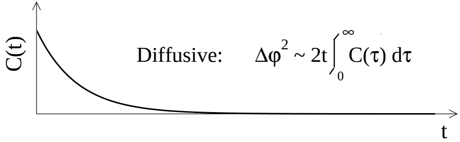
$\mathrm{Var}\,\hat{\varphi}(t) = \langle \hat{\varphi}(t)^2 \rangle - \langle \hat{\varphi}(t) \rangle^2$ where $\hat{\varphi}(t) = \hat{\theta}(t) - \hat{\theta}(0)$

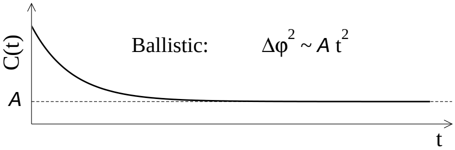
Phase spreading and correlation function

phase variance:

$$\mathrm{Var}\,\hat{\varphi}(t) = 2t\int_0^t C(\tau)d\tau - 2\int_0^t \tau C(\tau)d\tau$$

where **correlation function:** $C(t) = \langle\dot{\hat{\varphi}}(t)\dot{\hat{\varphi}}(0)\rangle - \langle\dot{\hat{\varphi}}(t)\rangle\langle\dot{\hat{\varphi}}(0)\rangle$





How to calculate $C(t)$?

phase derivative: $\dot{\hat{\varphi}} \simeq -\frac{\mu}{\hbar} - \sum_{\mathbf{k}\neq\mathbf{0}} A_{\mathbf{k}} \hat{n}_{\mathbf{k}}$,
where $\hat{n}_{\mathbf{k}}$ are occupation numbers of Bogoliubov modes

$$C(t) = \sum_{\mathbf{k}} \sum_{\mathbf{k}'} A_{\mathbf{k}} A_{\mathbf{k}'} \langle \delta n_{\mathbf{k}}(t) \delta n_{\mathbf{k}'}(0) \rangle = \vec{A}\,\vec{x}$$

where

$$\delta n_{\mathbf{k}}(t) = n_{\mathbf{k}}(t) - \bar{n}_{\mathbf{k}}$$

and $A_{\mathbf{k}} = \frac{g}{\hbar V}(U_{\mathbf{k}} + V_{\mathbf{k}})^2$, $x_{\mathbf{k}} = \sum_{\mathbf{k}'} A_{\mathbf{k}'} \langle \delta n_{\mathbf{k}}(t) \delta n_{\mathbf{k}'}(0) \rangle$

Description of mode occupation numbers is needed!

Kinetic equation

$$\dot{n}_{\mathbf{k}} = -\Gamma_{\mathbf{k}} n_{\mathbf{k}} + I(n_{\mathbf{k}})$$

where damping $\Gamma_{\mathbf{k}}$ and gain $I(n_{\mathbf{k}})$ rates are calculated within Bogoliubov method by using Fermi golden rule.

Important aspects:

- condensate treated as a isolated quantum system
- Landau and Beliaev processes included: $k + q \rightarrow q'$ and $k \rightarrow q + q'$
- energy is strictly conserved in each scattering event
- stationary solution: $\bar{n}_{\mathbf{k}} = \frac{1}{e^{\beta \epsilon_{\mathbf{k}}} - 1}$

Analytical result

Asymptotic expression for the condensate phase variance:

$$\mathrm{Var}\,\hat{\varphi}(t) \simeq \mathcal{A}t^2 + \mathcal{B}t + \mathcal{C} \qquad \text{for} \qquad t \to \infty$$

with:

$$\mathcal{A} = \frac{1}{\hbar^2}\left(\frac{d\mu}{dE}\right)^2_{E=\bar{E}} \mathrm{Var}\,E$$

$$\mathcal{B} = -2\vec{A}\,M^{-1}\,\vec{X}(0) \qquad \text{and} \qquad \mathcal{C} = -2\vec{A}\,M^{-2}\,\vec{X}(0)$$

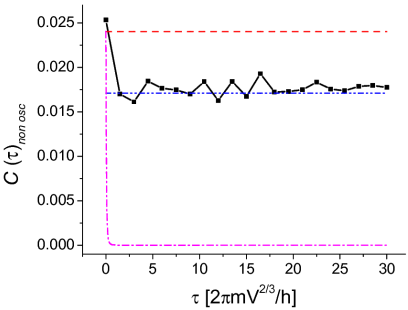
- we found explicit expressions for $\mathcal{B}$ and $\mathcal{C}$ in terms of M and $t=0$ correlation functions
- $\mathcal{B}$ and $\mathcal{C}$ do not depend on energy fluctuations in the initial state
- $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ depend on temperature

A. Sinatra, Y. Castin, E. Witkowska, Phys.Rev.A **80**, 033614 (2009)

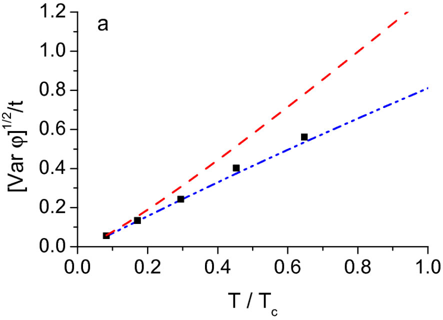
Ballistic spreading of the phase

Classical counterpart of the analytical treatment (blue lines) against classical fields simulations (points) **for varied temperature**.

$$C(t) \to \mathcal{A}_{\text{cl}}$$



$$\frac{\sqrt{\text{Var}\,\varphi(t)}}{t} \simeq \sqrt{\mathcal{A}_{\text{cl}}}$$

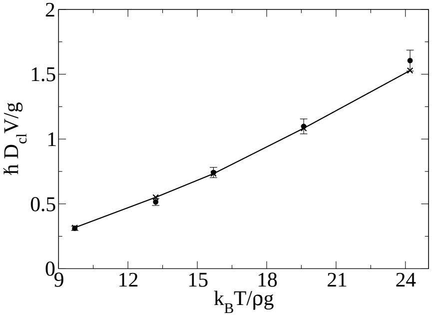


A. Sinatra, Y. Castin, E. Witkowska, Phys.Rev.A **75**, 033616 (2007)

Phase Diffusion

Classical counterpart of the analytical treatment (×) against classical fields simulations (•) **for varied temperature**.

$$\mathrm{Var}\,\varphi(t) \simeq 2D_{\mathrm{cl}}\,t$$



A. Sinatra, Y. Castin, Phys.Rev.A **78**, 053615 (2008)

A. Sinatra, Y. Castin, E. Witkowska, Phys.Rev.A **80**, 033614 (2009)

Conclusions

- using kinetic equations we calculated how the variance of the phase grows in time for a nonzero temperature:

$$\mathrm{Var}\,\hat{\varphi}(t) \simeq \mathcal{A}t^2 + \mathcal{B}t + \mathcal{C} \qquad \text{for} \qquad t \to \infty$$

- ballistic spreading of the phase is present within canonical distribution of the initial states: $\mathcal{A} \propto \mathrm{Var}\,E$
- after suppression of the energy fluctuations phase undergo diffusion motion with $D = \mathcal{B}/2$
- classical fields simulations as a guideline and test of predictions of the quantum theory
- thermal effect on the phase spreading should be accessible to experiments with present technology