Bose-Einstein condensates and Fock states: classical or quantum?

All the nice (quantum) things that a simple beam splitter can do!

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« Theory of Quantum Gases and Quantum Coherence »
Nice, 2-4 June 2010

Nice, le 02/06/2010

1. Schrödinger and his wave function; real or not real?

Fock states with high populations: is the wave function a classical field?

2. Anderson and his phase; spontaneous symmetry breaking in superfluids

3. A single beam splitter. Classical phase and quantum angle. Generalized Hong-Ou-Mandel effect

4. Interference experiments with beam splitters

4.1 Population oscillations

4.2 Creating NOON states, Leggett's QSMDS (quantum superpositions of macroscopically distinct states)

4.3 Violating the Bell inequalities (BCHSH) with a double interference experiment

5. Spin condensates: Einstein-Podolsky-Rosen ; Anderson's phase = hidden variable.

Nice, le 02/06/2010

1. Schrödinger and his wave function

- The prehistory of quantum mechanics: Bohr's quantized trajectories, quantum jumps, Heisenberg's matrix mechanics

- The undulatory period: Schrödinger. The world is made of waves, which propagate in configuration space

- The standard/Copenhagen interpretation: the wave function is a tool to calculate probabilities; it does not directly represent reality.

Limitations of the wave function

- With a single quantum system, as soon as the wave function is measured, it suddenly changes (state reduction).
- One cannot perform exclusive measurements on the same system (Bohr's complementarity)
- One cannot determine the wave function of a single system perfectly well (but one can teleport it without knowing it)
- One cannot clone the wave function of a single quantum system

But, if you have many particles with the same wave function (quantum state), these limitations do not apply anymore. The wave function becomes similar to a classical field. You can use some particles to make one measurements, the others to make a complementary (exclusive) measurement.

Bose-Einstein condensation (BEC)

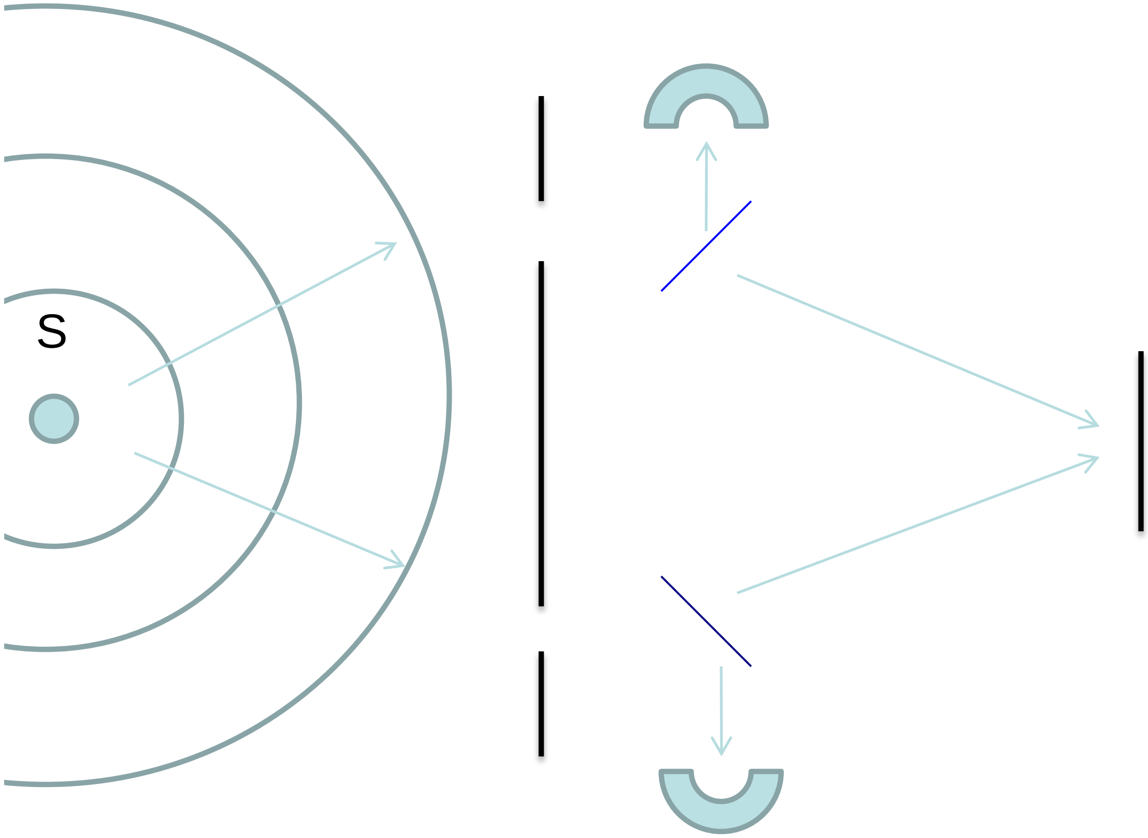
- BEC can be achieved in dilute gases

- It provides a mechanism to put an arbitrary number of particles into the same quantum state; the repulsive interactions stabilize the condensate

- The wave function becomes a (complex) macroscopic classical field

- When many particles occupy the same quantum state, one can use some of them to make one kind of measurement, others to make complementary measurements (impossible with a single particle).

Complementary measurements



The wave function of a Bose-Einstein condensate looks classical

- One can see photographs (of its squared modulus)

- One can take little pieces of the wave function and make them interfere with each other (one the sees the effects of the phase)

- One cas see the vibration modes of this field

- (limitations: thermal excitations; « particles above condensate »)

- A BE condensate looks very much like a classical field!

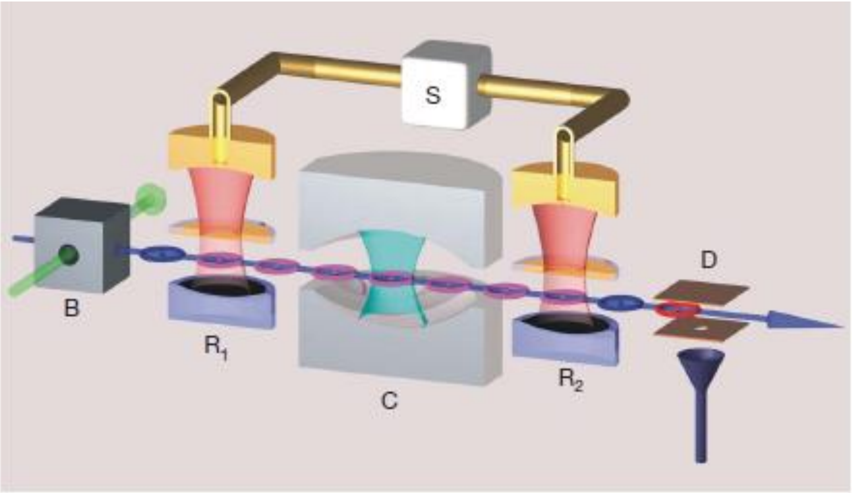
…..but not quite, as we will see

Other methods to populate Fock states

- Bose-Einstein condensation in dilute gases

- Continuous measurements of photons in cavities (Haroche, Raimond, Brune et al.).

Measuring the number of photons in a cavity



Quantum jumps of light recording the birth and death of a photon in a cavity

Sébastien Gleyzes[1], Stefan Kuhr[1]†, Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Ulrich Busk Hoff[1], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

Nature, vol 446, mars 2007

Measuring the number of photons in a cavity (2)

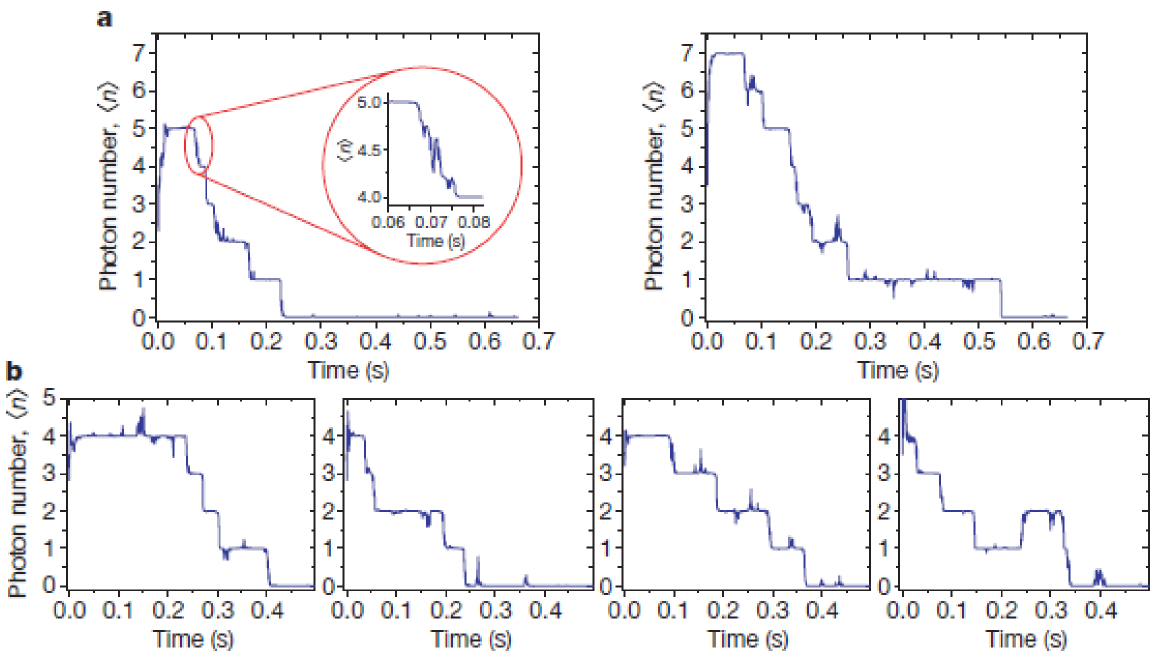


Figure 4 | Repeated QND measurements. **a**, Mean photon number $\langle n \rangle$ followed over 0.7 s for the two sequences whose collapse is analysed in Fig. 2. After converging, $\langle n \rangle$ remains steady for a while, before successive quantum jumps bring it down to vacuum. Inset, zoom into the $n = 5$ to 4 jump, showing that it is detected in a time of ~0.01 s. **b**, Four other signals recording the evolution of $\langle n \rangle$ after field collapse into $n = 4$. Note in the leftmost frame the exceptionally long-lived $n = 4$ state, and in the rightmost frame the $n = 1$ to 2 jump revealing a thermal field fluctuation.

Progressive field-state collapse and quantum non-demolition photon counting

Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Clément Sayrin[1], Sébastien Gleyzes[1], Stefan Kuhr[1]†, Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

Nature, vol 448, 23 August 2007

Continous quantum non-demolition measurement and quantum feedback (1)

PRL 97, 073601 (2006) PHYSICAL REVIEW LETTERS week ending 18 AUGUST 2006

Deterministic and Nondestructively Verifiable Preparation of Photon Number States

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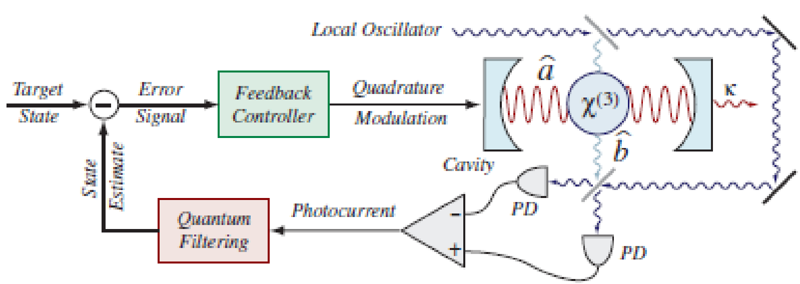
An experimentally viable approach for preparing arbitrary photon number states of a cavity mode using continuous measurement and real-time quantum feedback is presented. The procedure passively monitors the number state actually achieved in each feedback-stabilized measurement trajectory, thus providing nondestructively verifiable photon generation. The feasibility of a possible cavity QED implementation in the many-atom, good-cavity-coupling regime is analyzed.



Continous quantum non-demolition measurement and quantum feedback (2)

PHYSICAL REVIEW A **80**, 013805 (2009)

Quantum feedback by discrete quantum nondemolition measurements: Towards on-demand generation of photon-number states

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(Received 1 May 2009; published 9 July 2009)

We propose a quantum feedback scheme for the preparation and protection of photon-number states of light trapped in a high-Q microwave cavity. A quantum nondemolition measurement of the c
information on the photon-number distribution. The feedback loop is closed by injectin
coherent pulse adjusted to increase the probability of the target photon number. The effici
of the closed-loop state stabilization is assessed by quantum Monte Carlo simulations.
realistic experimental conditions, the Fock states are efficiently produced and protected aı

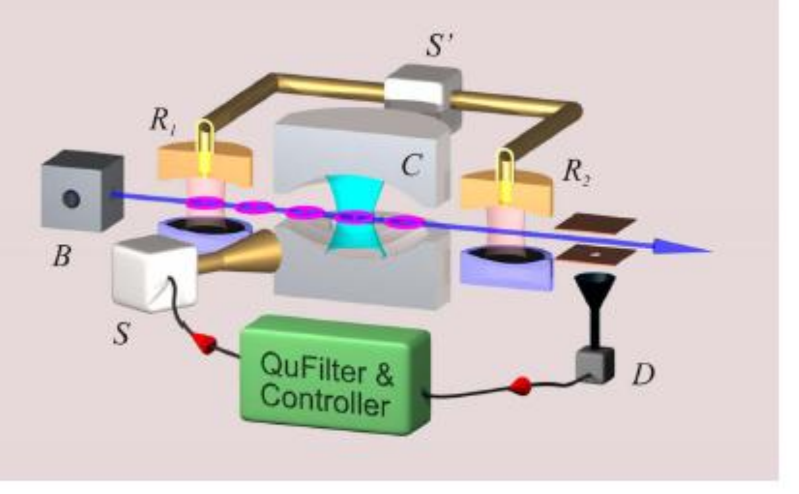


FIG. 1. (Color online) Proposed quantum feedback scheme adapted to a microwave cavity QED setup. C: high-Q microwave

2. Anderson's phase (1966)

Spontaneous symmetry breaking in superfluids

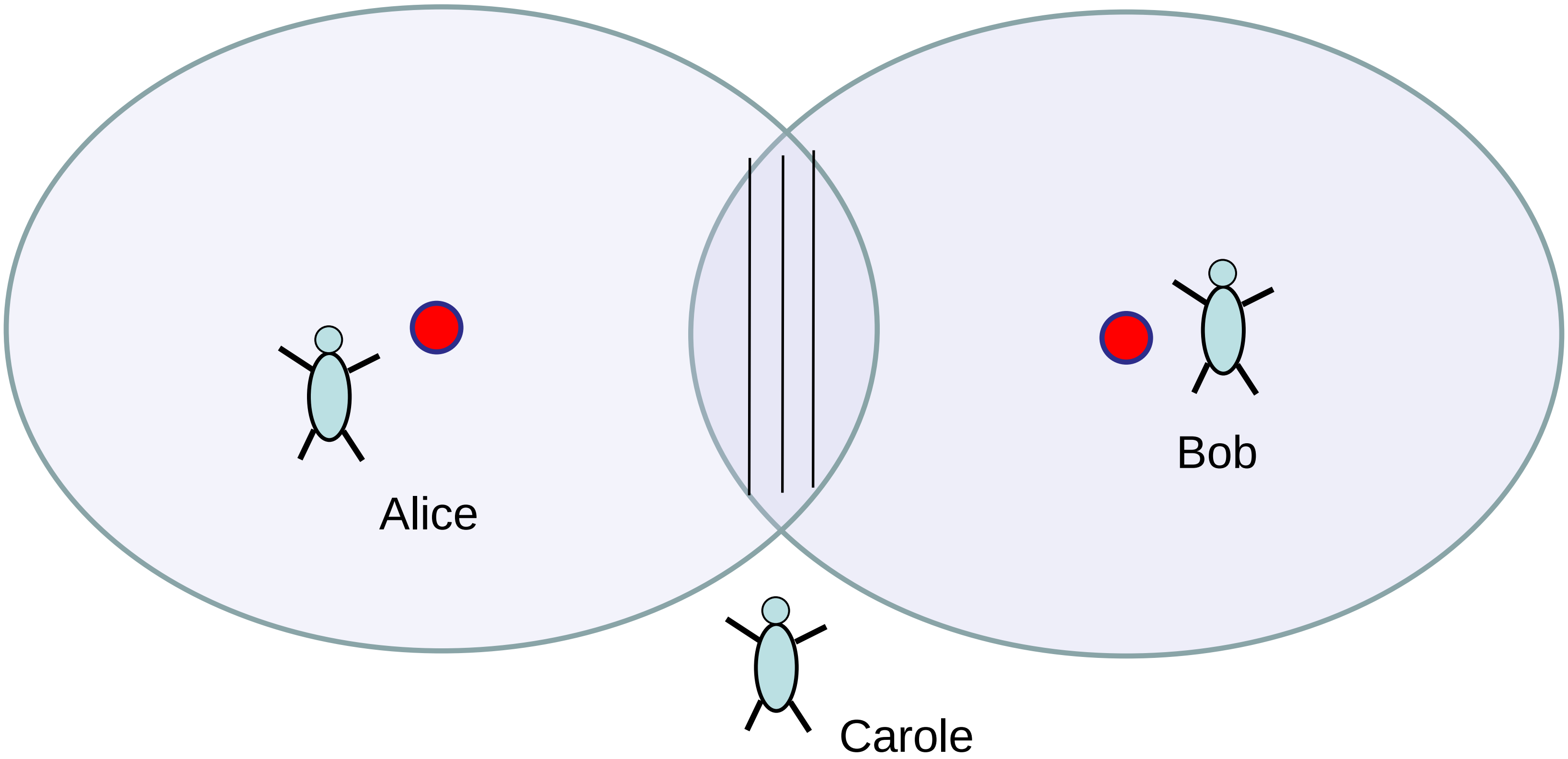
• **When a system of bosons undergoes the superfluid transition (BEC), spontaneous symmetry breaking takes place; the order parameter is the macroscopic wave function $<\Psi>$, which takes a non-zero value. This creates a (complex) classical field with a classical phase. Similar to ferromagnetic transition.**

• **Very powerful idea! It naturally explains superfluid currents, vortex quantization, etc..**

• **Violation of the conservation of the number of particles, spontaneous symmetry breaking; no physical mechanism.**

• **Anderson's question: "When two superfluids that have never seen each other before overlap, do they have a (relative) phase?"**

Relative phase of two condensates in quantum mechanics (spinless condensates)



Experiment: interferences between two independent condensates

M.R. Andrews, C.G. Townsend, H.J. Miesner, D.S. Durfee, D.M. Kurn and W. Ketterle, Science **275**, 637 (1997).

It seems that the answer to Anderson's question is « yes ». The phase takes completely random values from one realization of the experiment to the next, but remains consistent with the choice of a single value for a single experiment.

Interference beween condensates without spontaneous symmetry breaking

- J. Javanainen and Sun Mi Ho, "Quantum phase of a Bose-Einstein condensate with an arbitrary number of atoms", Phys. Rev. Lett. 76, 161-164 (1996).
- T. Wong, M.J. Collett and D.F. Walls, "Interference of two Bose-Einstein condensates with collisions", Phys. Rev. A 54, R3718-3721 (1996)
- J.I. Cirac, C.W. Gardiner, M. Naraschewski and P. Zoller, "Continuous observation of interference fringes from Bose condensates", Phys. Rev. A 54, R3714-3717 (1996).
- Y. Castin and J. Dalibard, "Relative phase of two Bose-Einstein condensates", Phys. Rev. A 55, 4330-4337 (1997)
- K. Mølmer, "Optical coherence: a convenient fiction", Phys. Rev. A 55, 3195-3203 (1997).
- K. Mølmer, "Quantum entanglement and classical behaviour", J. Mod. Opt. 44, 1937-1956 (1997)
-C. Cohen-Tannoudji, Collège de France 1999-2000 lectures, chap. V et VI "Emergence d'une phase relative sous l'effet des processus de détection" http://www.phys.ens.fr/cours/college-de-france/.
- etc.

How Bose-Einstein condensates acquire a phase under the effect of successive quantum measurements

Initial state before measurement: $$|N_\alpha N_\beta\rangle = \frac{1}{\sqrt{N_\alpha! N_\beta!}} a_\alpha^{\dagger N_\alpha} a_\beta^{\dagger N_\beta} |0\rangle$$

No phase at all ! This state contains N particles;
no number fluctuation => no phase.

One then measures the positions $\mathbf{r}_1$, $\mathbf{r}_2$, etc. of the particles. M measurements are performed.

If M<<N = $N_\alpha + N_\beta$, the combined probability for the M measurements is:

$$P(\mathbf{r}_1, \cdots \mathbf{r}_M) \sim \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} \prod_{i=1}^{M} [1 + \cos(\mathbf{k}\cdot\mathbf{r}_i + \lambda)]$$

Emergence of the (relative) phase under the effect of quantum measurement

• For a given realization of the experiment, while more and more particles are measured, the phase distribution becomes narrower and narrower; in other words, the Anderson phase did not exist initially, but emerges progressively and becomes better and better defined.

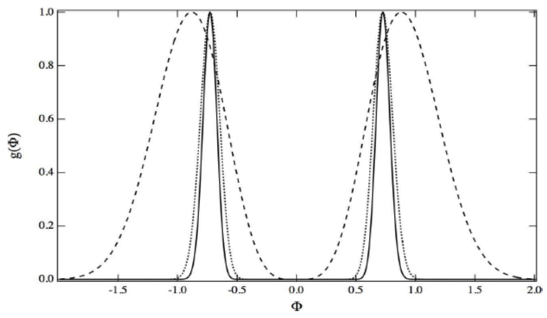


FIG. 2: The angular distribution $g(\Phi)$ as a function of angle for three iteration step lengths, 10 steps (dashed line), 150 steps (dotted line), and 300 steps (solid line). For a single measuring angle this always has two equal peaks corresponding to the intersection of the spin cone with the transverse plane. The peaks narrow with step length.

• For another realization, the value chosen by the phase is different

• If the experiment is repeated many times, the phase average reconstructs the semi-classical results (curves that are flat in the center, and raise on the sides). One then recovers all results of the Anderson theory.

The phase is similar to a « hidden variable »

An additional (or « hidden ») variable λ (the relative phase) appears very naturally in the calculation, within perfectly orthodox quantum mechanics. Ironically, mathematically it appears as a consequence of the number conservation rule, not of its violation!

F. Laloë, “The hidden phase of Fock states; quantum non-local effects”, European Physical Journal **33**, 87-97 (2005).

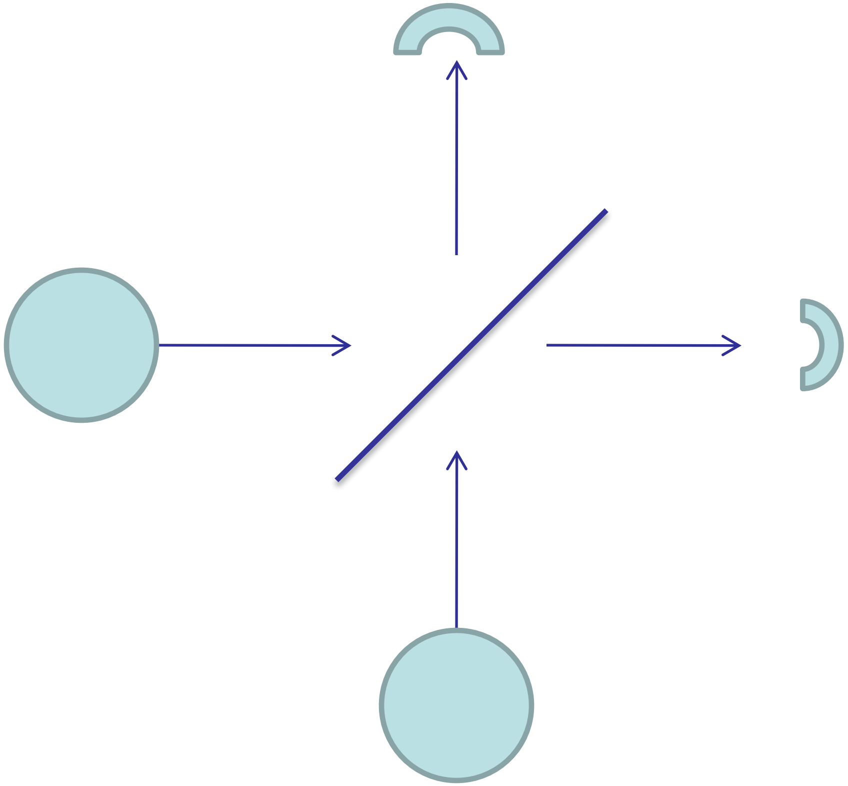
Is Anderson's classical phase equivalent to an ab initio quantum calculation?

In the preceding calculation, using Anderson's phase or doing an ab initio calculation is a matter of preference; the final results are the same.

Is this a general rule? Is the phase always classical?

Actually, no! We now discuss several examples which are beyond a simple treatment with symmetry breaking, and illustrate really quantum properties of the (relative) phase of two condensates.

3. A single beam splitter



Classical optics

$$r = \frac{2x}{1+x^2} = \frac{2\sqrt{I_\alpha I_\beta}}{I_\alpha + I_\beta} \leq 1$$

$$P_{semi-class.}(m_1, m_2) = \frac{N!}{2^N \; m_1! m_2!} \left[1 + r\cos\left(\lambda - \pi/2\right)\right]^{m_1} \left[1 - r\cos\left(\lambda - \pi/2\right)\right]^{m_2} \quad (4)$$

For Fock states, we expect that the relative phase λ should be completely random, so that this expression becomes:

$$P^{Fock}_{semi-class.}(m_1, m_2) = \frac{N!}{2^N \; m_1! m_2!} \int_{-\pi}^{+\pi} \frac{d\lambda}{2\pi} \; \left[1 + r\cos\lambda\right]^{m_1} \left[1 - r\cos\lambda\right]^{m_2} \quad (5)$$

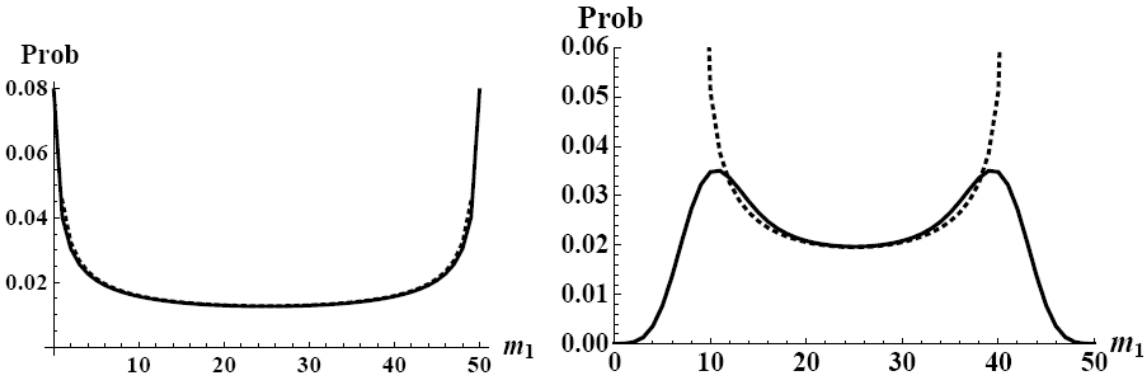


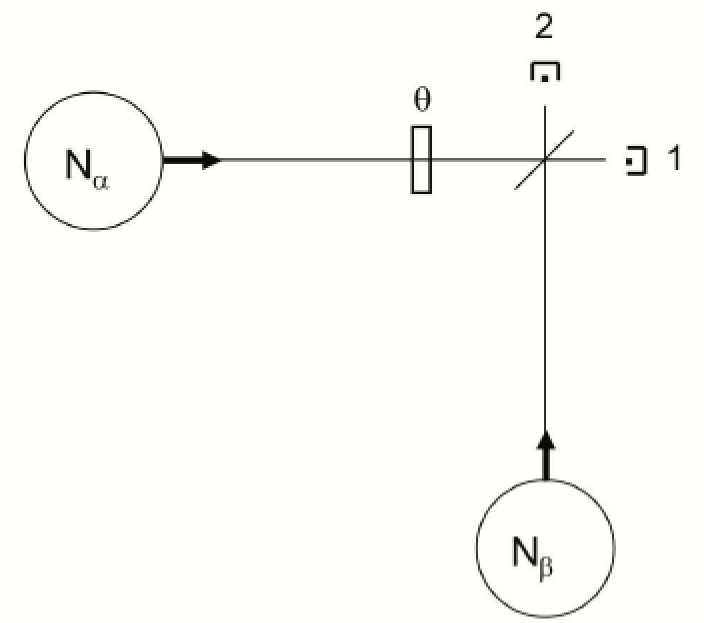
Figure 2: The left part (a) shows the classical (dotted line) and semiclassical (full line) distributions as a function of m_1 when the intensities of the input beams are equal ($x = r = 1$) and when $m_1 + m_2 = 50$. Values of m_1 near the maximum and the minimum are more likely to occur with this distribution. The right part (b) shows the same distributions for the same total number of particles, but when the intensities of the input beams are different (their ratio is 6/44).

Quantum mechanics

- Hong-Ou-Mandel effect (HOM); two input photons, one on each side; they always leave in the same direction (never in two different directions).

- Generalization: arbitrary numbers of particles N_α et N_β in the sources

$$|N_\alpha, N_\beta\rangle = \frac{1}{\sqrt{N_\alpha! N_\beta!}} a_\alpha^{\dagger N_\alpha} a_\beta^{\dagger N_\beta} |0\rangle$$



With BE condensates, one can obtain the equivalent of beam splitters by Bragg reflecting the condensates on the interference pattern of two lasers, and observe interference effects (see e.g. W. Phillips and coworkers)

Quantum calculation

$$P(m_1, m_2) = \frac{N_\alpha! N_\beta!}{m_1! m_2!} \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} \int_{-\pi}^{\pi} \frac{d\Lambda}{2\pi} \cos\left[(N_\alpha - N_\beta)\Lambda\right] \left[\cos\Lambda + \cos\lambda\right]^{m_1} \left[\cos\Lambda - \cos\lambda\right]^{m_2}$$

Two angles appear, the classical phase λ and the quantum angle Λ.

• If only some particles are missed, a $[\cos \Lambda]^{N-M}$ appears inside the integral, where N is the total number of particles, and M the number of measured particles.

If **Λ=0,** one recovers the classical formula

• The quantum angle plays a role when all particles are measured. It contains properties that are beyond the classical phase (Anderson's phase). It is the source of the HOM effect for instance

Measuring all particles

The quantum angle Λ plays an important role

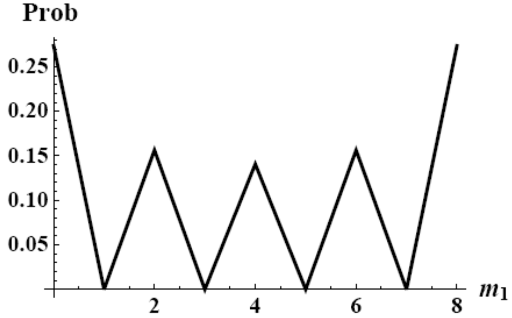


Figure 4: The probability for $N_\alpha = 4$, $N_\beta = 4$ illustrating the rule that, if an even number of particles enters each side of the beam splitter, an even number must emerge from each side.

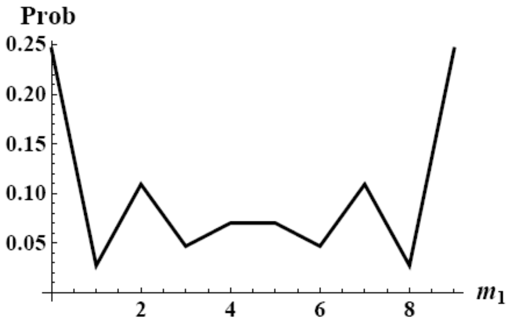


Figure 5: The probability for $N_\alpha = 4$, $N_\beta = 5$. The probability no longer vanishes for odd m_1 but oscillations remain. Now, of course, odd m_1 implies even m_2 and vice versa.

Other examples

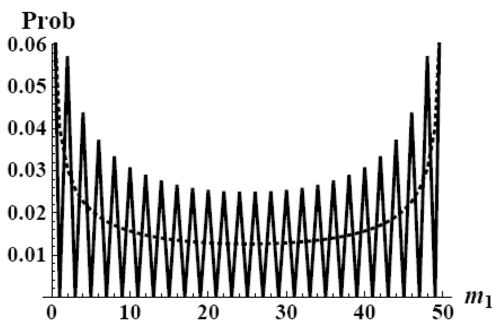


Figure 6: The probability for $N_\alpha = N_\beta = 25$. The probability vanishes for odd m_1. The graph has been cut off at $m_1 = 0, 50$ where it is about twice as high. The variation with $1/\sqrt{m_1 m_2}$ is evident. The dotted line shows the corresponding semi-classical distribution

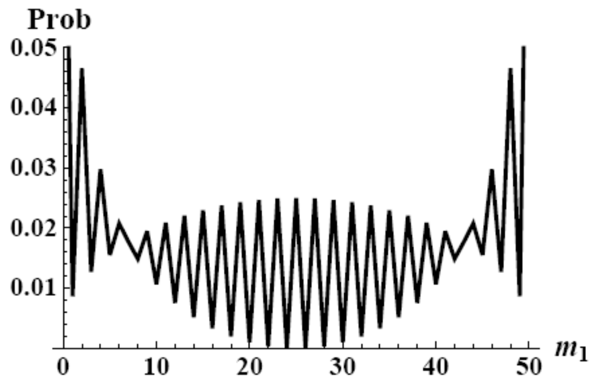


Figure 8: The probability for $N_\alpha = 26$, $N_\beta = 24$.

F. Laloë et W. Mullin, Festschrift en l'honneur de H. Rauch et D. Greenberger (Vienne, 2009)

Neither Anderson, nor HOM .. but both combined

Repeating the HOM experiment many times:

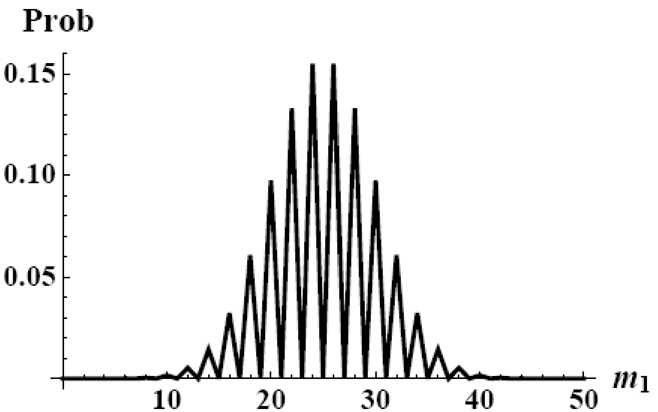


Figure 10: The pair probability for $N_\alpha = 25,\ N_\beta = 25$.

Populating Fock states:

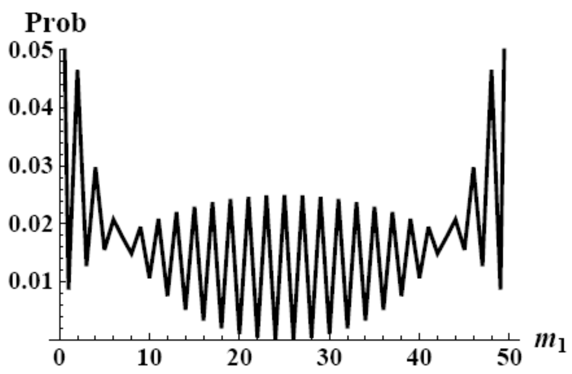


Figure 8: The probability for $N_\alpha = 26,\ N_\beta = 24$.

The result looks completely different. The photons tend to spontaneously acquire a relative phase in the two channels under the effect of quantum measurement.

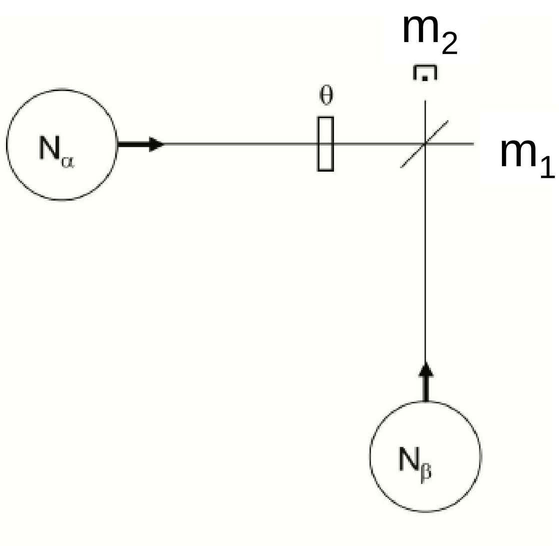
4. Experiments with more beam splitters

4.1 Population oscillations

4.2 Creating NOON states

4.3 Double interference experiment, Bell violations, quantum non-locality with Fock states

Appearance of the phase



It it impossible to know from which input beam the detected particles originate. After measurement, the number of particles in each input beam fluctuates, and their relative phase becomes known.

A phase state

- If m_1 (or m_2) =0, the measurement process determines the relative phase between the two input beams

- After a few measurements, one reaches a « phase state »:

$$|\Phi\rangle = \left[a_\alpha^\dagger + e^{i\Phi} a_\beta^\dagger\right]^{N-M} |\text{vacuum}\rangle = \sum_{P=0}^{N-M} \frac{(N-M)!}{P!(N-M-P)!} e^{iP\Phi} \left[a_\alpha^\dagger\right]^{N-M-P} \left[a_\beta^\dagger\right]^{P} |\text{vacuum}\rangle$$

- The number of particles in each beam fluctuates

A macroscopic quantum superposition

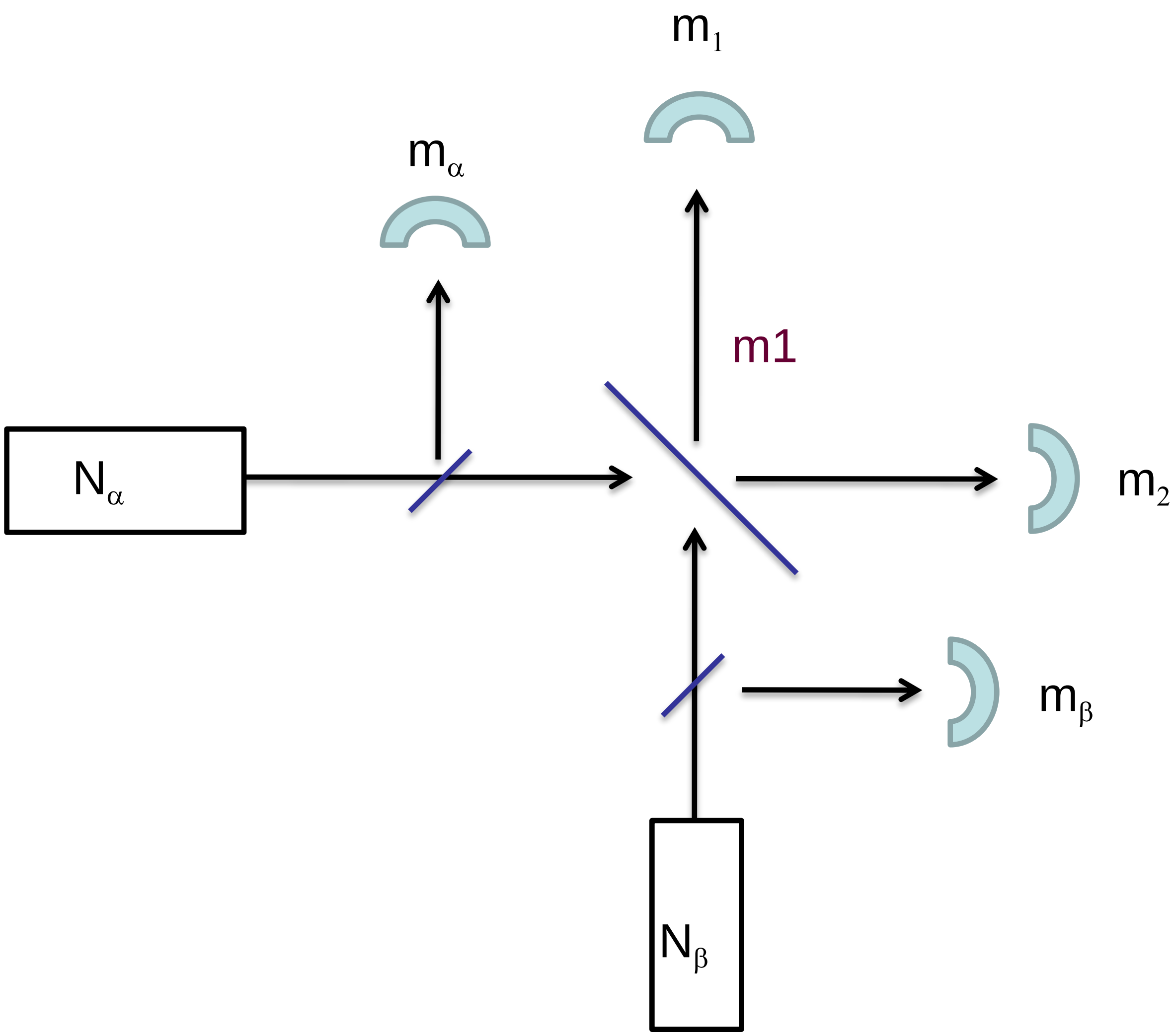
• If $m_1 = m_2$, the measurement process does not select one possible value for the relative phase, but two at the same time.

• This creates a quantum superposition of two phase states:

$$|\Psi\rangle = |\Phi\rangle + |-\Phi\rangle =$$
$$= \sum_{P=0}^{N-M} \frac{(N-M)!}{P!(N-M-P)!} \left(e^{iP\Phi} + e^{-iP\Phi}\right) \left[a_\alpha^\dagger\right]^{N-M-P} \left[a_\beta^\dagger\right]^P |\text{vacuum}\rangle$$

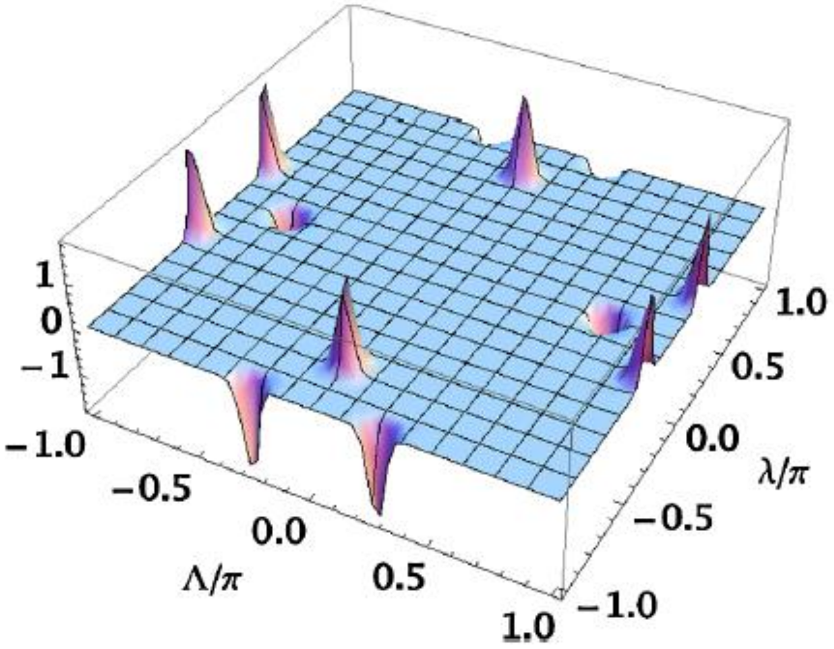
Possibility of oscillations

4. 1 Populations oscillations



Quantum calculation

$$P(m_1,m_2,m_\alpha,m_\beta)=\frac{N_\alpha!N_\beta!}{m_1!m_2!m_\alpha!m_\beta!2^N}\int_{-\pi}^{\pi}\frac{d\Lambda}{2\pi}\cos[(N_\alpha-m_\alpha-N_\beta+m_\beta)\Lambda]\int_{-\pi}^{\pi}\frac{d\lambda}{2\pi}[\cos\Lambda+\cos\lambda]^{m_1}[\cos\Lambda-\cos\lambda]^{m_2}.$$



W.J. Mullin and F. Laloë, PRL **104**, 150401 (2010)

Detecting the quantum superposition

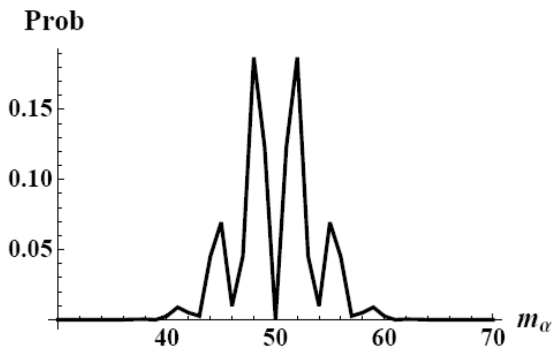


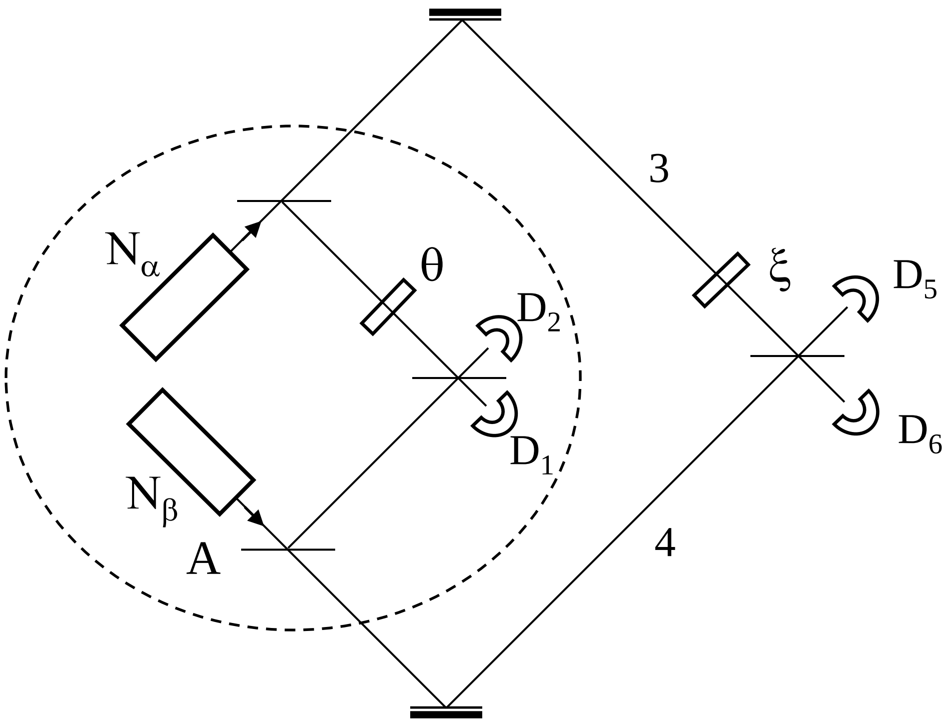
FIG. 4: Plot of $P(m_1, m_\alpha)$ given by Eq. (16) versus m_α for $N_\alpha = N_\beta = M = 100$, $m_1 = 17$ and $m_2 = 83$. If m_2 is even, the central dip is replaced by a peak.

The measurement process creates fluctuations of the number of particles in each input beam.

On sees oscillations in the populations, directly at the output of the particle sources.

4.2 Creating NOON states

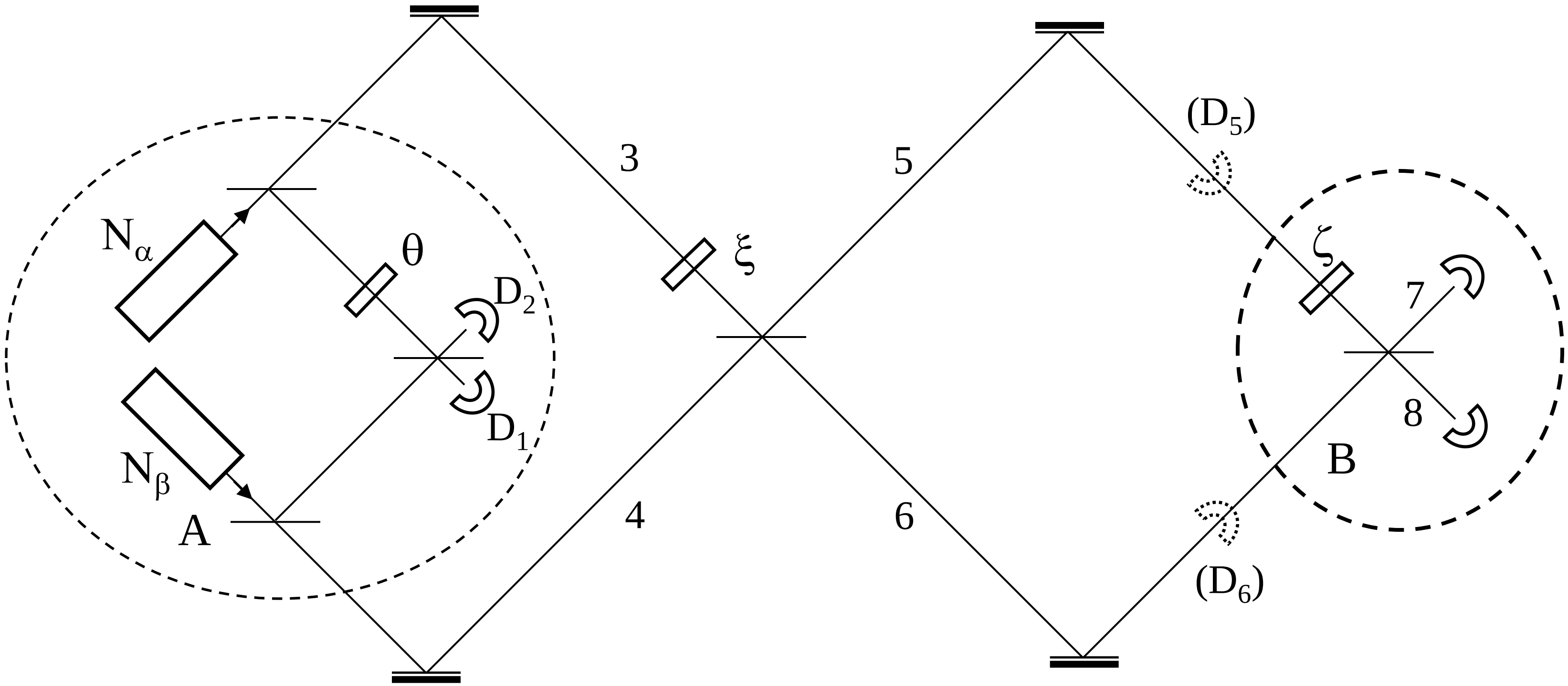
$$|\Psi\rangle \sim \left[\left[a_\alpha^\dagger\right]^N + \left[a_\beta^\dagger\right]^N \right] |\text{vacuum}\rangle \sim |N,0\rangle + |0,N\rangle$$



Création of a « NOON state » in arms 5 and 6

Leggett’s QSMDS

(quantum superpositions of macroscopically different states)



Detection in arms 7 and 8 of the NOON state in arms 5 and 6

4.3 Non-local quantum effects

Testing how to BEC's spontaneously choose a relative phase in two remote places

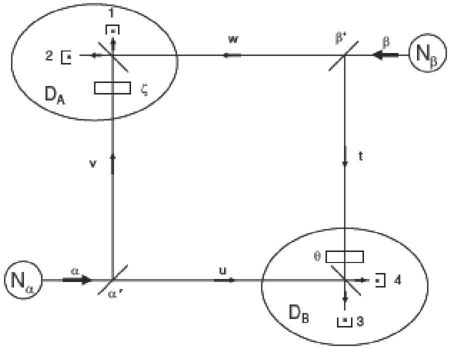


Figure 1: Two Fock states, with populations N_α and N_β, enter beam splitters, and are then made to interfere in two different regions of space D_A and D_B, with detectors 1 and 2 in the former, 3 and 4 in the latter. The number of particles m_j in each of the channels $j = 1, 2, 3, 4$ are counted.

Alice measures m_1 and m_2, Bob measures m_3 and m_4.

Both choose to measure the observed parity: $A=(-1)^{m_1}$; $B=(-1)^{m_2}$

Violating the BCHSH inequalities

$$Q = \langle \mathcal{A}\mathcal{B}\rangle + \langle \mathcal{A}\mathcal{B}'\rangle + \langle \mathcal{A}'\mathcal{B}\rangle - \langle \mathcal{A}'\mathcal{B}'\rangle$$

Classically:

$$-2 \leq AB + AB' \pm (A'B - A'B') \leq 2$$

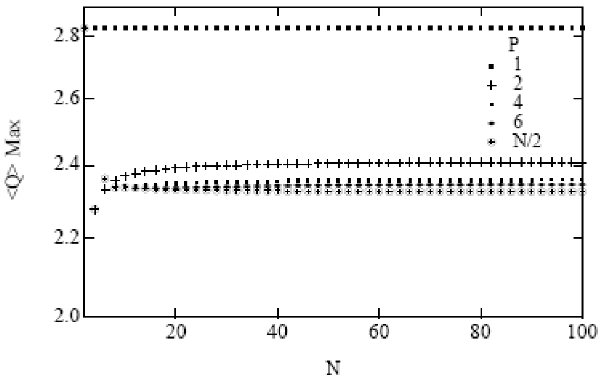


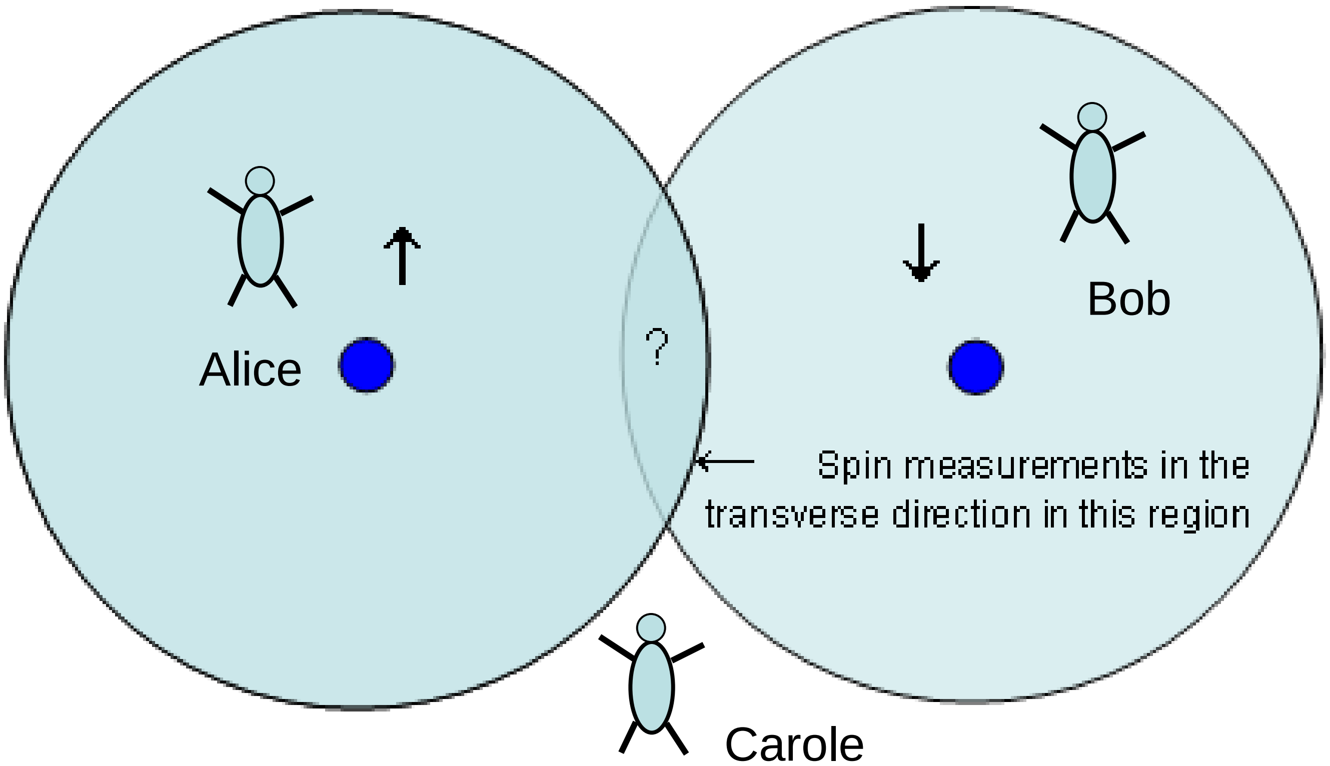
FIG. 1: The maximum of the quantum average $< Q >$ for Alice doing P experiments and Bob $N - P$, as a function of the total number of particles N. The usual Bell situation is obtained for $N = 2, P = 1$. Local realist theories predict an upper limit of 2; large violations of this limit are obtained, even with macroscopic systems ($N \to \infty$). If $P = 1$, the violation saturates the Cirel'son limit for any N.

On predicts strong violations of the Bell inequanlities, even if the total number of particles is large.

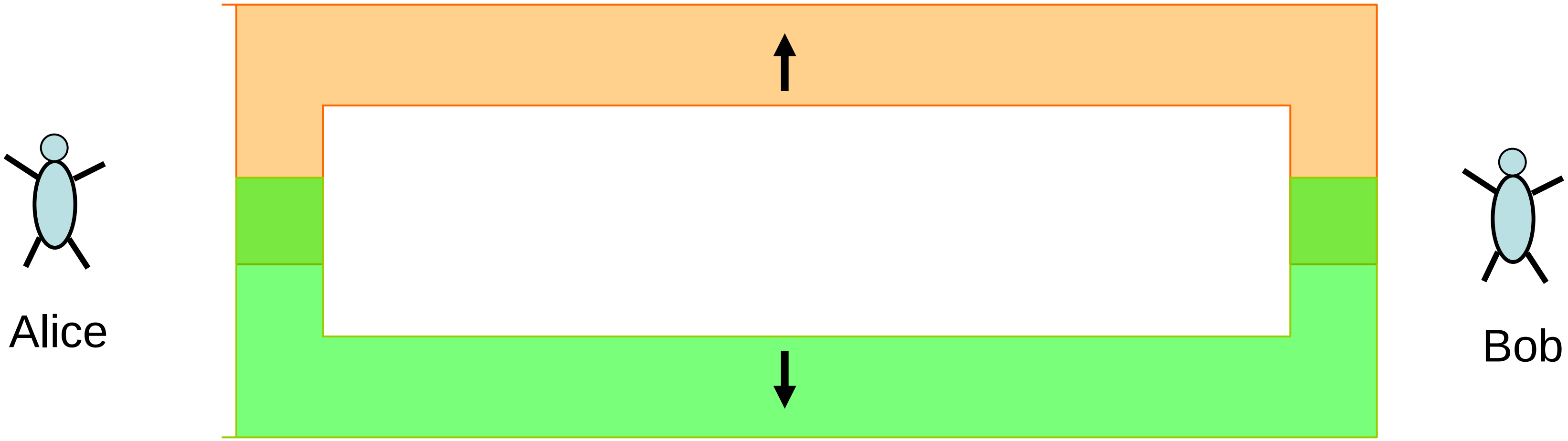
5. The EPR argument with spin condensates

$$|\Phi_0\rangle = |N_+ : \varphi, + \; ; \; N_- : \varphi', -\rangle = \left[a^\dagger_{\varphi,+}\right]^{N_+} \left[a^\dagger_{\varphi',-}\right]^{N_-} |\text{vacuum}\rangle$$

$$| \Phi_0 > \; = \; | N_a : \varphi_a, \uparrow ; N_b : \varphi_b, \downarrow \; >$$



EPR argument



Orthodox quantum mechanics tells us that it is the measurement performed by Alice that creates the transverse orientation observed by Bob.

It is just the relative phase of the mathematical wave functions that is determined by measurements; the physical states themselves remain unchanged; nothing physical propagates along the condensates, Bogolobov phonons for instance, etc.

EPR argument: the « elements of reality » contained in Bob's region of space can not change under the effect of a measurement performed in Alice's arbitrarily remote region. They necessarily pre-exited; therefore quantum mechanics is incomplete.

Agreement between Einstein and Anderson.

But this is precisely what the spontaneous symmetry breaking argument is saying! the relative phase existed before the measurement, as soon as the condensates were formed.

So, in this case, Anderson’s phase appears as a macroscopic version of the « EPR element of reality », applied to the case of relative phases of two condensates. It is an additional variable, a « hidden variable » (Bohm, etc.).

Bohr's reply to the usual EPR argument (with two microscopic particles)

The notion of physical reality used by EPR is ambiguous; it does not apply to the microscopic world; it can only be defined in the context of a precise experiment involving macroscopic measurement apparatuses.

But here, the transverse spin orientation may be macroscopic! We do not know what Bohr would have replied to the BEC version of the EPR argument.

Conclusion

- Many quantum effects are possible with Fock states
- The wave function of highly populated quantum state (BEC's) has classical properties, but also retains strong quantum features
- One needs to control the populations of the states. A possibility: small BEC's, stabilization by repulsive interactions
- Or optics: non-linear generation of photons (parametric downconversion), or continuous quantum measurement